

Introduction to swaps

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incorporating ideas from

“Teaching interest rate and currency swaps”

by Keith C. Brown (Texas-Austin)

and Donald J. Smith (Boston U.)

Swaps: contracts specifying
exchange of cash flows:
fixed price traded for floating price
fixed rate traded for floating rate

Swap structures include:
interest rate swaps
currency swaps
commodity swaps
vast assortment of variations
quanto, basis, index differential, etc.

Interest rate swap

exchange of fixed rate for a floating reference rate,
or exchange of one variable rate for another.

periodic net settlements made by comparing rates,
adjusting for day-count conventions, then
multiplying rate difference by notional principal:

Settlement amount = notional principal times:

$$\left(\text{Fixed rate} \times \frac{\# \text{days}}{\text{year}} \right) - \left(\text{Floating rate} \times \frac{\# \text{days}}{\text{year}} \right)$$

where **#days** and **year** are determined by day-count convention

Day-count conventions:

The number of days assumed to be in a month and in a year when calculating the interest accrued on a debt instrument.

Treasury notes, bonds: actual over actual

Corporate bonds: 30 over 360

money-market

(**Libor**): actual over 360 (usual)

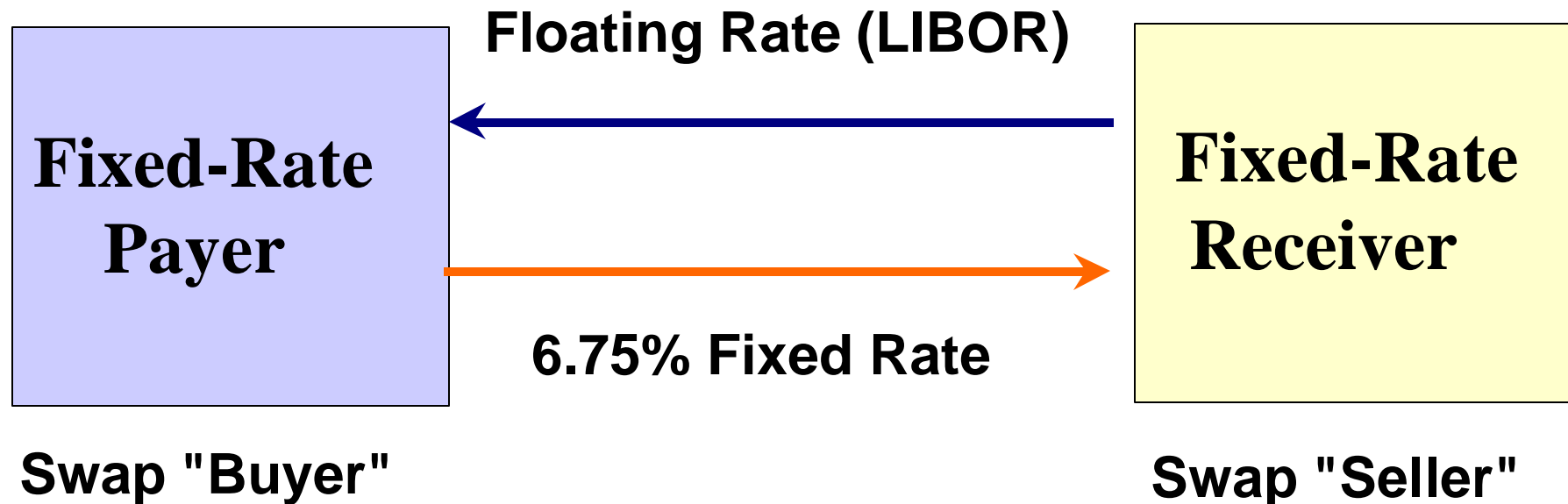
actual days is actual number of days in period

actual for years is either 365 or 366.

Interest rate swap "Buyer" and "Seller"

payer of fixed rate is "buyer".

Fixed rate payer "buys" floating rate (LIBOR),
the fixed rate is the "price"



Currency swaps

exchange fixed (or floating) cash flows
denominated in one currency for fixed (or floating)
cash flows denominated in another currency.

Examples:

- fixed \$ for fixed DM

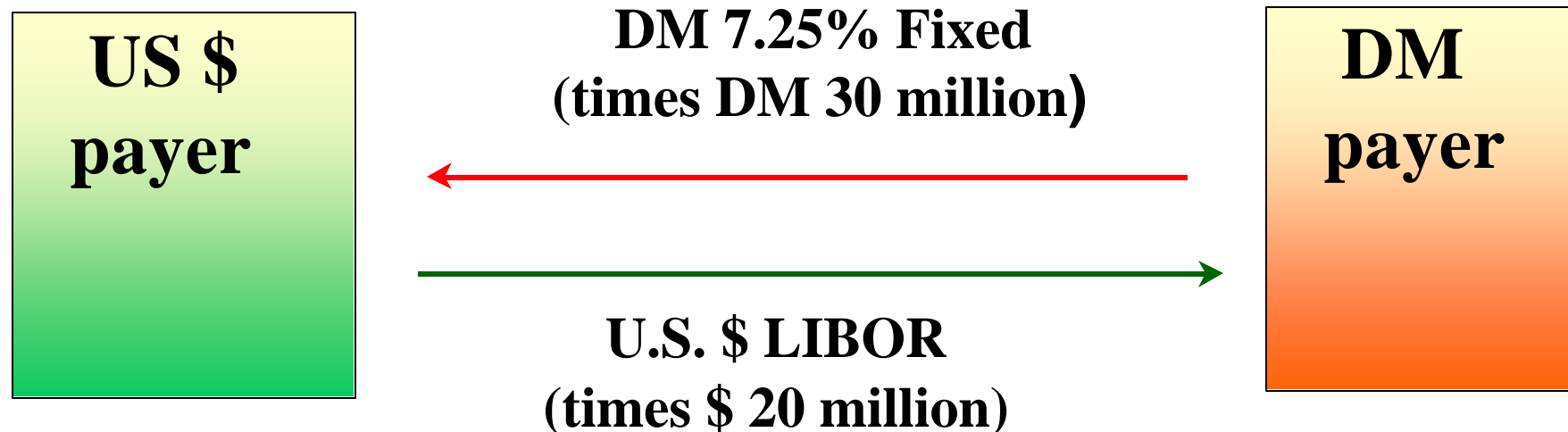
- fixed \$ for floating DM

- floating \$ for fixed DM

- floating \$ for floating DM

Usually principal is exchanged at origination
and at maturity.

Fixed/floating \$/DM swap



settlement dates: April 11, October 11 of each year

rate set 2 days prior. payment: \$ is actual/360; DM is 30/360

Oct. 9, 1998, \$ LIBOR is 5.5%.

Assume payment is in arrears on April 11, 1999:

US\$ payer owes \$556,111.11 (= .055 x 182/360 x \$20 million)

DM payer owes DM 1,087,500 (= .0725 x 180/360 x DM 30 mm)

Settlement on either gross basis

or net based on spot \$/DM exchange rate on 4/11/99.

Swap Applications

New Issue "Arbitrage"

(lower borrowing costs)

Access new markets

lower borrowing costs (borrower)

increase returns (investor)

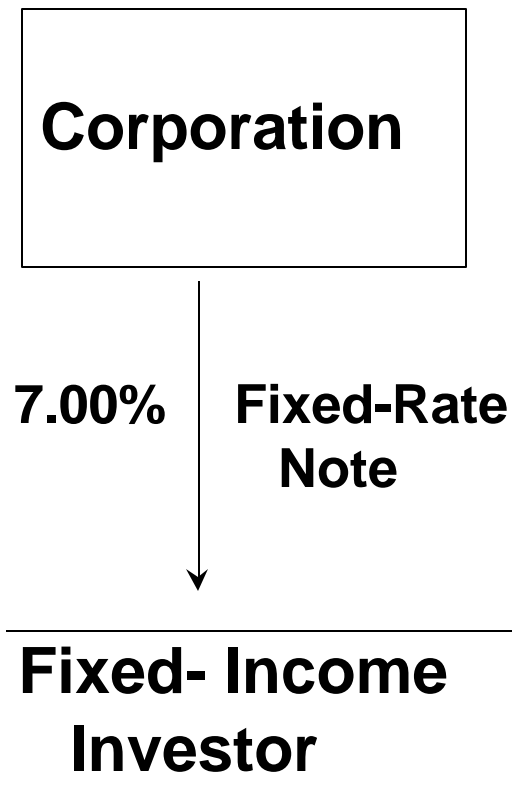
Financial restructuring (transform risks)

Hedge exposures

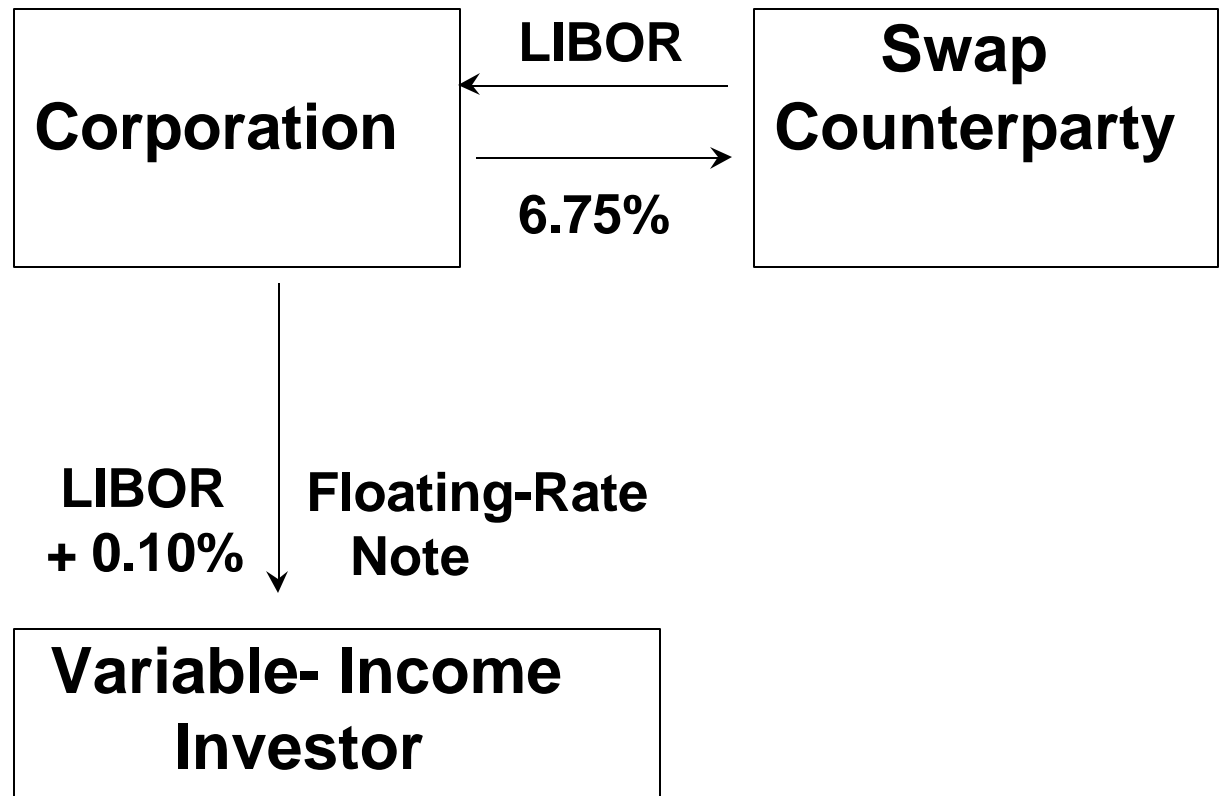
create synthetic portfolios

New issue "arbitrage"

Straight Debt



Swap-Driven Debt Structure



Cost of Funds with swap structure: 6.85%

Comments on "New issue arbitrage"

Is the swap structure comparable to straight debt?
not if straight debt is callable - straight debt
rate is higher as payment for call option.

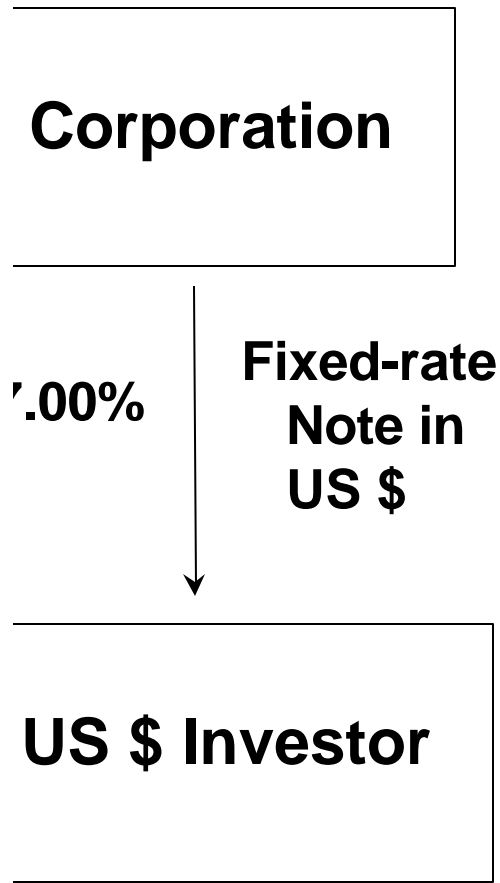
Swap structures may have embedded options
(credit triggers, e.g. Texaco & Banker's Trust)

Swap structure should provide lower cost of funds,
as swap contains counterparty credit risk:
joint probability of default and replacement
swap with higher fixed rate.

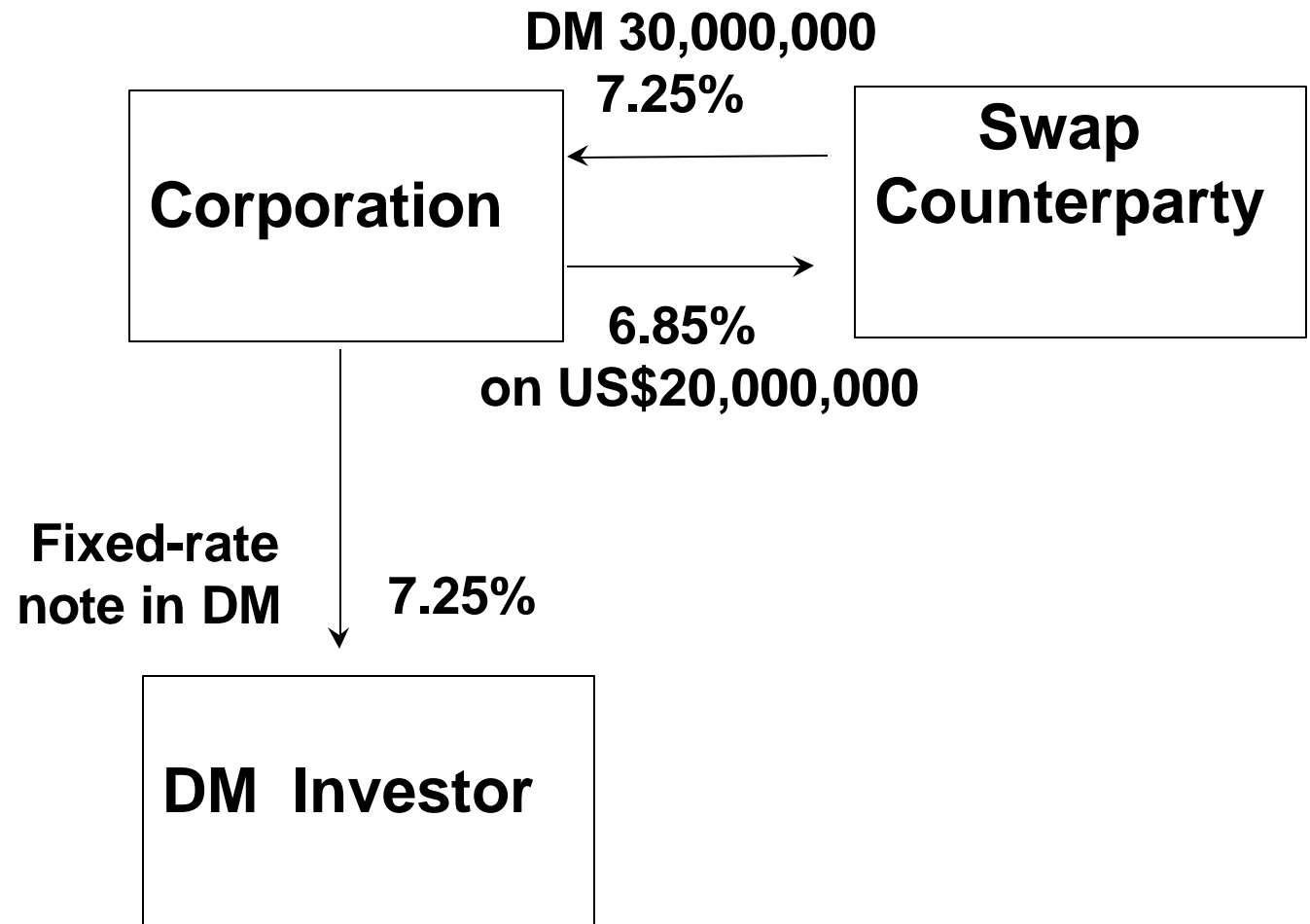
Tax and accounting conventions may matter also.

Swaps to access new markets

Straight Debt



Swap-Driven Debt Structure



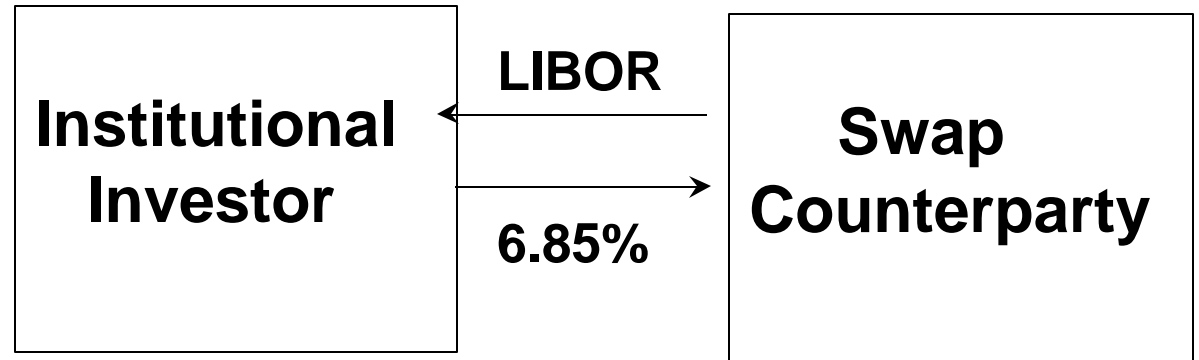
Cost of Funds with swap structure: 6.85%

Swaps to restructure debt

Straight Debt



Swap-Driven Debt Structure



Asset return with swap structure: LIBOR + 1.15%

Interpretations of swap contracts

1. Series of Forward Contracts

useful to understand initial pricing

interest rate swap is series of FRAs

currency swap is series of FX forwards

2. Combination of Bonds

useful for mark-to-market

insights from duration and convexity

3. Combination of Options

multiperiod put-call parity

risk management insights

Forward Rate Agreements (FRAs)

FRA is **one-date** interest rate swap
usually LIBOR for fixed.
cash-settled, OTC forward contract
(no margin account, mark-to market)

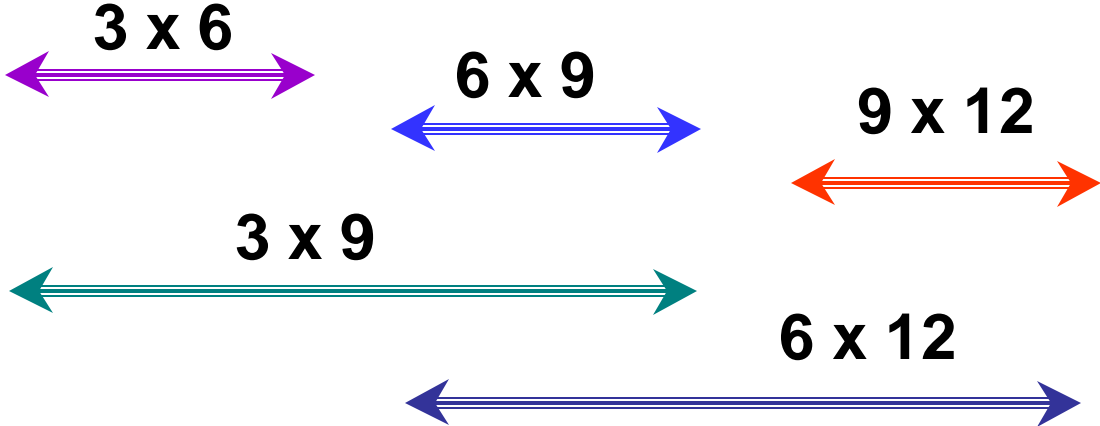
Settlement and maturity dates part of FRA name:

- **3x6** FRA is **3-month** LIBOR, **3 months** forward
- **6x9** FRA is **3-month** LIBOR, **6 months** forward
- **6x12** FRA is **6-month** LIBOR, **6 months** forward

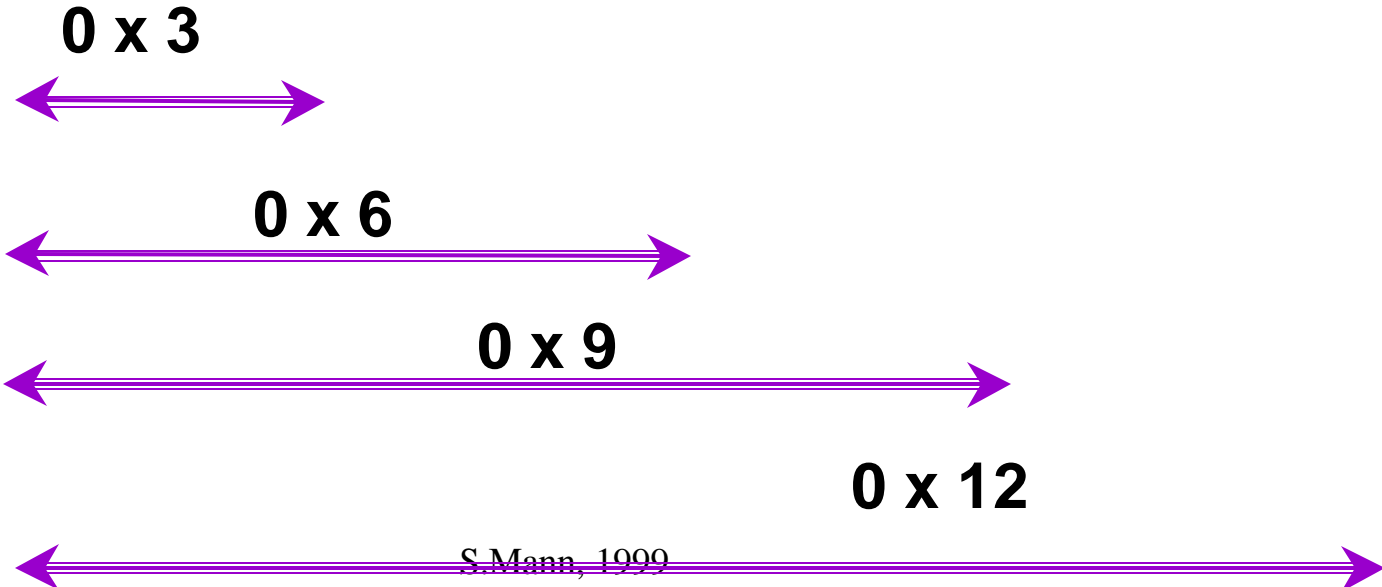
FRAs

Dates **0** **3** **6** **9** **12**

**FRAs at
Date 0**



**Cash
Market at
Date 0**



FRA payoffs

Payoff to **long** FRA holder (**pays fixed**) is:

(Notional Principal) times:

$$(\text{LIBOR} - \text{FRA rate}) \times (\text{days}/360)$$

Example: Notional principal = \$10,000,000

3 x 6 FRA : 3 month forward, 90 day FRA

FRA is 7.00 today.

Assume that 3 months later, 3 month LIBOR is 7.10%

Then Payoff to **long** is:

$$\$10 \text{ million} \times (.0710 - .0700)(90/360) = \$2500$$

Payoff per million on any forward 90 day FRA (X x 3)

is \$25 per basis point change in forward LIBOR

Pricing FRAs

0 x 3 rate is 5.00 % ; 0 x 6 rate is 5.25 %

What is 3 x 6 FRA rate?

Unless $(1 + r_{(0x6)})^{.5} = (1 + r_{(0x3)})^{.25}(1 + r_{(3x6)})^{.25}$
there is arbitrage.

So

$$r_{(3 \times 6)} = \left\{ \frac{(1 + r_{(0x6)})^{.5}}{(1 + r_{(0x3)})^{.25}} \right\}^4 - 1$$

Thus $r_{(3x6)} = 5.55006 \%$

\$ FRAs usually priced off Eurodollar futures

Swaps versus series of FRAs

Series of FRAs: **each FRA** will likely have a **different rate** (fixed payer pays different rates) according to the shape of the spot yield curve.

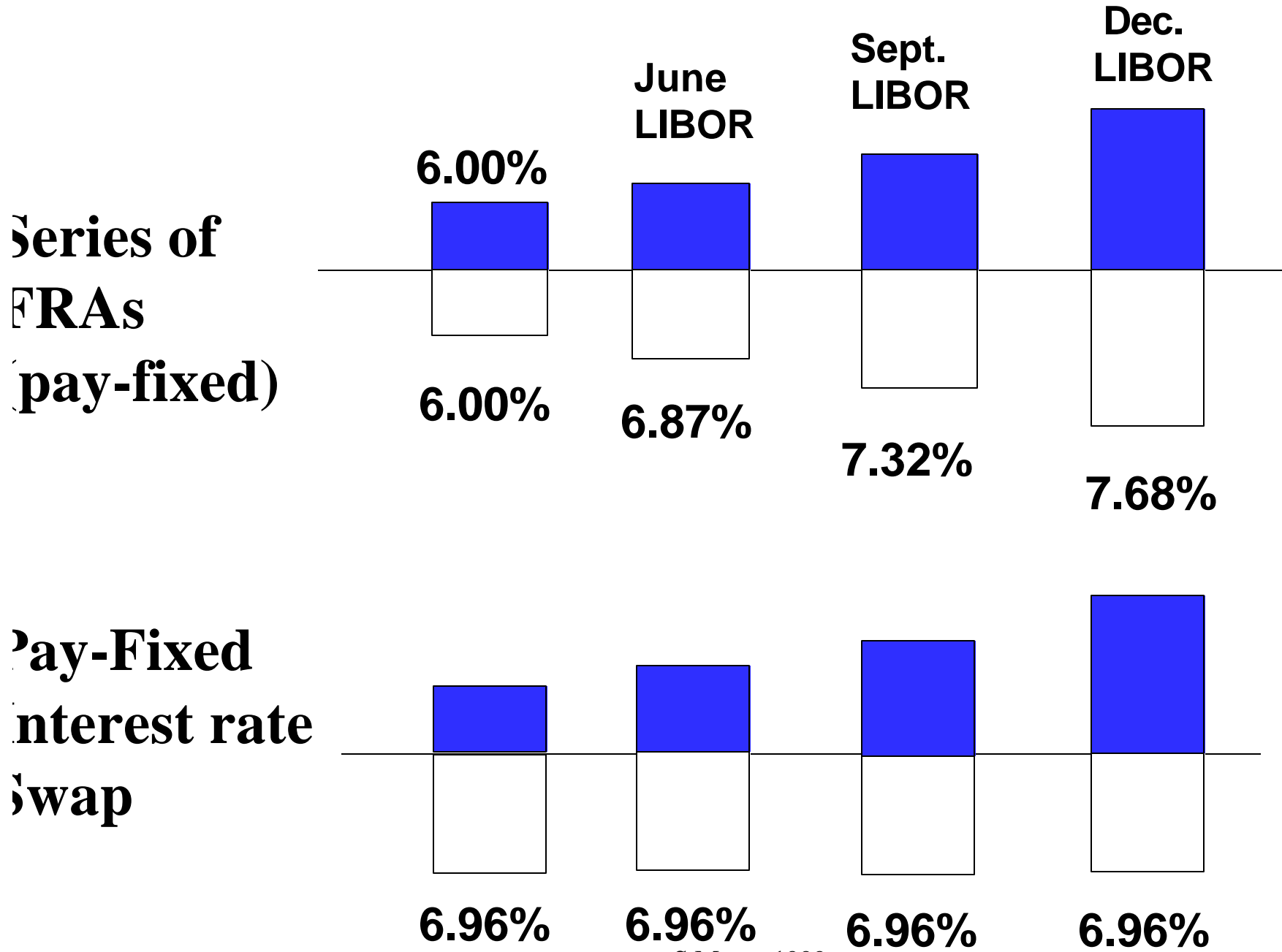
Swap: usually has the **same fixed rate** for all settlement periods.

At Origination:

Each FRA has **zero** economic value

Swap has zero value, but some settlement dates have positive value and some negative

Swap vs. FRA series



Pricing Interest rate swaps

swap fixed rate found by treating the swap as a series of "off market" FRAs, and setting the swap price such that the present value of the "off market" FRAs is equal to zero.

The June (3 x 6) FRA rate is 6.87%.

Let SFR be the Swap Fixed Rate,

Present value of the off market June FRA is:

$$\frac{(.0687 - \text{SFR})}{(1 + r_{(0 \times 3)})^{.25} (1 + r_{(3 \times 6)})^{.25}} \times (\text{Notional principal}) \times (90/360)$$

"Off market" FRA values

June swap leg value ("off market" FRA value):

$$\frac{(.0687 - \text{SFR})}{(1 + r_{(0 \times 3)})^{.25} (1 + r_{(3 \times 6)})^{.25}} \times (\text{Notional principal}) \times (90/360)$$

or

$$\frac{(.0687 - \text{SFR})}{(1.0600)^{.25} (1.0687)^{.25}} \times (\text{Notional principal}) \times (90/360)$$

Note that:

$$\frac{(.0687 - \text{SFR})}{(1.0600)^{.25} (1.0687)^{.25}} = \frac{(.0687 - \text{SFR})}{(1.064341)^{.50}} = \frac{(.0687 - \text{SFR})}{(1 + r_{(0 \times 6)})^{.50}}$$

S.Mann, 1999

Set PV of Swap legs equal to zero

$$0 = \frac{(.06 - \text{SFR})}{(1+r_{(0x3)})^{.25}} + \frac{(.0687 - \text{SFR})}{(1+r_{(0x6)})^{.50}} + \frac{(.0732 - \text{SFR})}{(1+r_{(0x9)})^{.75}} + \frac{(.0768 - \text{SFR})}{(1+r_{(0x12)})^{1.00}}$$

solve for SFR to find SFR = 6.96%

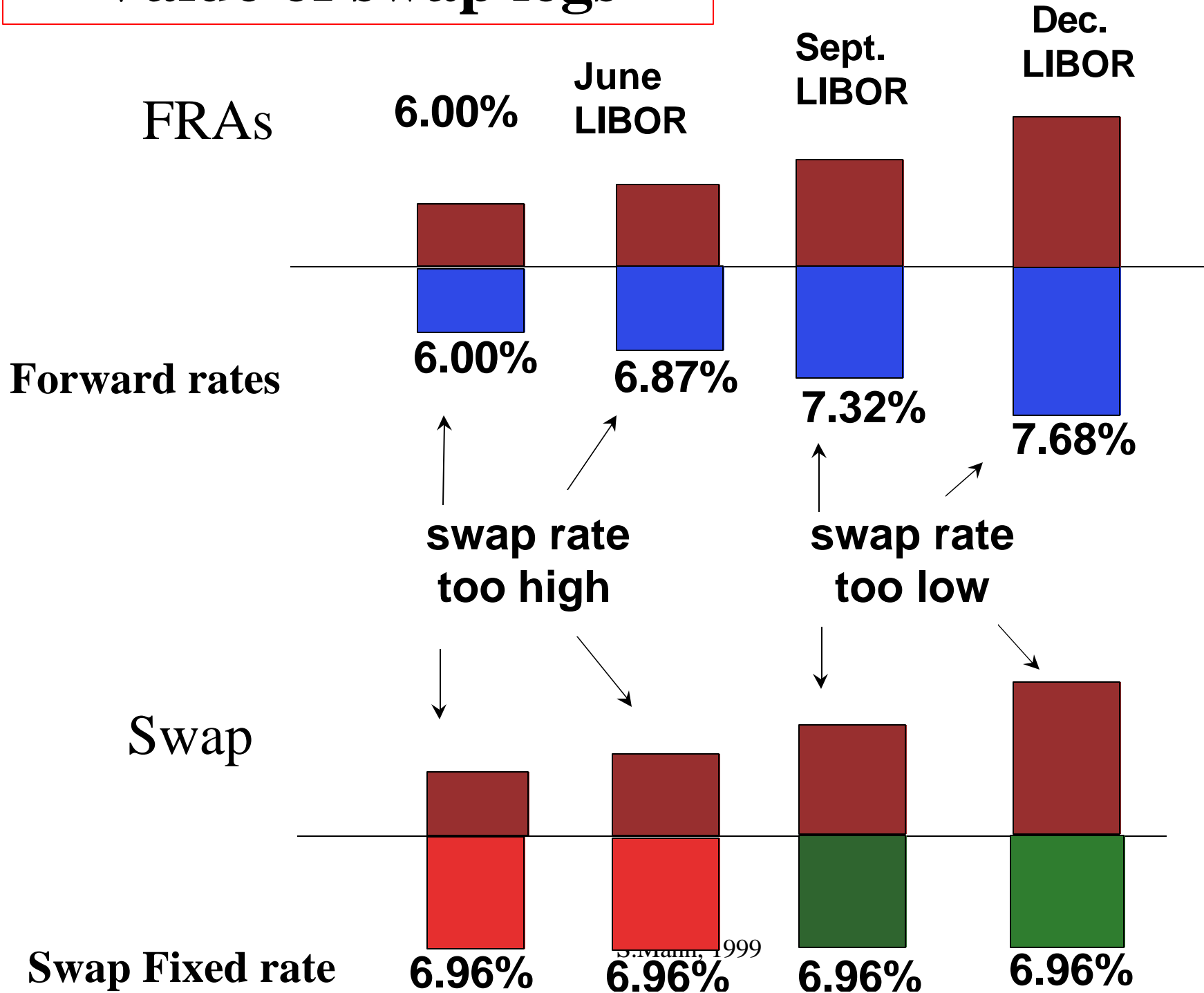
Note:

eliminated notional principal term (same multiple for each term)

each term is simply the current value of a forward contract:

value = PV (contract price - current forward price)

Value of swap legs



Swaps on a balance sheet

If swap pieces were placed on balance sheets:

Swap fixed payer

Swap fixed receiver

Assets

Liabilities

Assets

Liabilities

6 x 9	0 x 3
9 x 12	
	3 x 6

0 x 3	6 x 9
3 x 6	9 x 12

**credit risk is front-loaded for fixed-receiver,
back-loaded for fixed-payer
(if forward curve is upward sloping)**

Swaps after first settlement

Swap fixed payer

Assets

Liabilities

6 x 9	3 x 6
9 x 12	

swap becomes an asset
(unless rates
drop substantially)

Swap fixed receiver

Assets

Liabilities

3 x 6	6 x 9
	9 x 12

swap becomes a liability
(unless rates
drop substantially)

Swaps as Combination of Bonds

value swaps after origination (marking to market)
calculate swap duration and convexity

Example:

5-year swap: receive fixed 10%, pay LIBOR.
\$10 million Notional Principal with Semi-annual settlement

Net cash flows will be same as if corp had

- Buys 5-year, \$10 million, 10% coupon bond
- Issues \$10 million floating rate note (FRN)
at LIBOR flat

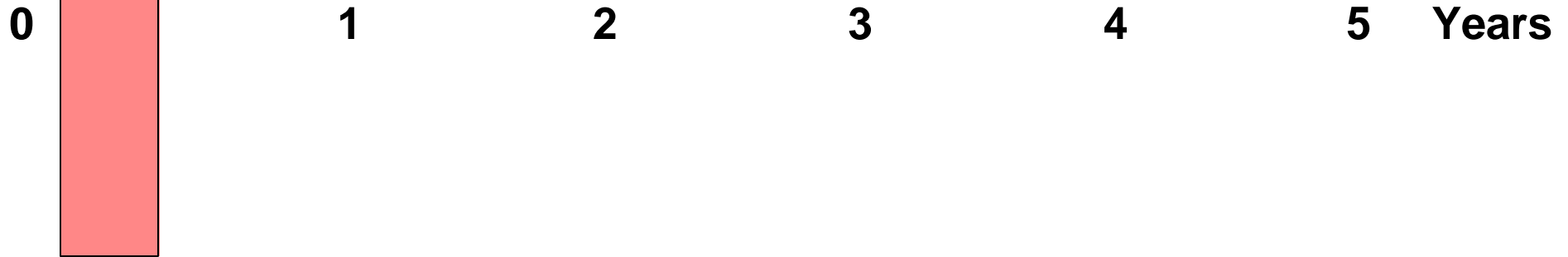
If bonds have same value, then swap is "at market"
otherwise swap is "off market" with initial payment

Swap as bond combination

Buy \$10 million, 10% fixed coupon, 5-year bond

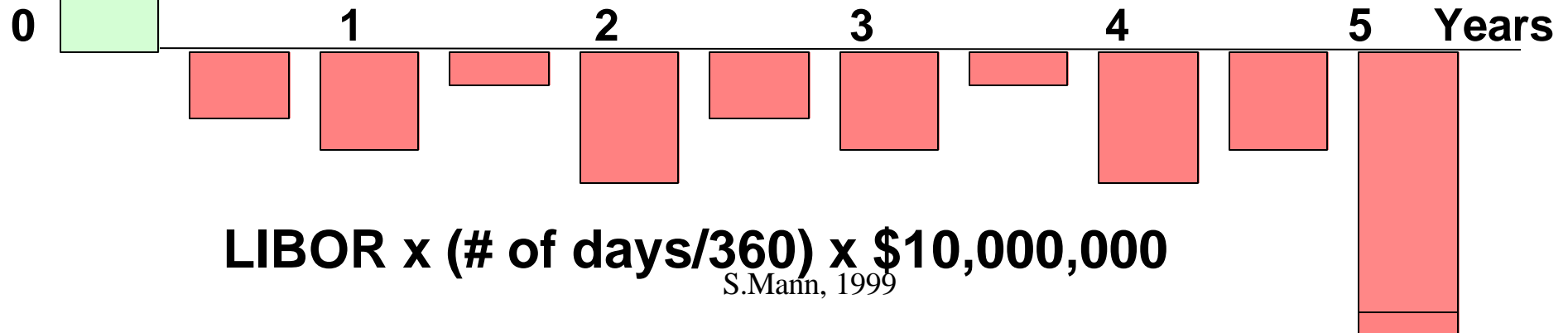
\$500,000 coupons

\$10,500,000



market value

Issue \$10 million, 5-year floating note at LIBOR flat

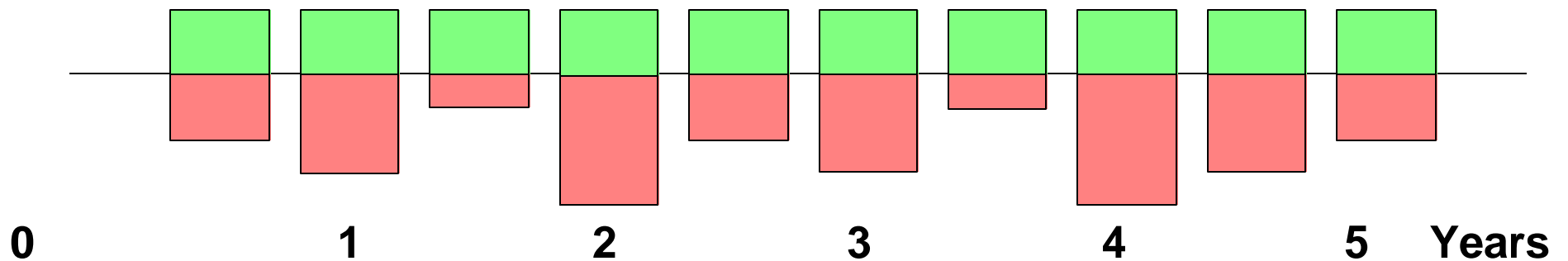


$\text{LIBOR} \times (\# \text{ of days}/360) \times \$10,000,000$

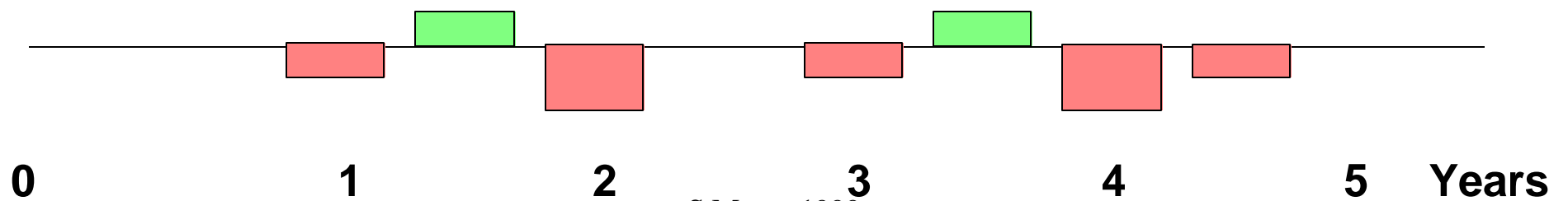
S.Mann, 1999

Swap as bond combination

Gross Settlement Flows on 10% versus LIBOR receive-fixed swap



Net Settlement Cash Flows



Swaps as bonds: Mark to market

At origination:

Swap fixed payer

Assets	Liabilities
Floater	Fixed Note

Swap fixed receiver

Assets	Liabilities
Fixed Note	Floater

mark to market is zero (if "on market")

If swap fixed rate rises:

Swap fixed payer

Assets	Liabilities
Floater	Fixed Note

Swap fixed receiver

Assets	Liabilities
Fixed Note	Floater

Swaps as Option Combinations

Cap (interest rate cap agreement)

- series of European, cash settled put options
- underlying asset is hypothetical debt security
 - Eurodollar time deposit for caps on LIBOR
- gains value as rates rise
 - (underlying asset price drops with rate increase)

Floor (interest rate floor agreement)

- series of European, cash settled call options
- underlying asset is hypothetical debt security
 - Eurodollar time deposit for LIBOR floors
- gains value as rates drop
 - (underlying asset price rises as rates fall)

Cap example

firm pays 250 basis points times Notional Principal
for a 5-year, 8% cap on six-month **LIBOR**
Cap buyer receives settlement payments
whenever **LIBOR** exceeds cap strike rate

The settlement payment:

if LIBOR \leq 8.00%

0

if LIBOR $>$ 8.00%

$(\text{LIBOR} - .08) \times (180/360) \times$
Notional Principal

example :

LIBOR is 8.50% at settlement, payment received
is $(.005)(1/2)(\text{NP}) = \2500 per \$1 million
cap owner receives \$50 per million
for each basis point above the strike

Floor example

firm pays 150 basis points times Notional Principal
for a 5-year, 4.50% floor on six-month **LIBOR**
Floor buyer receives settlement payments
whenever **LIBOR** is less than floor strike rate

The settlement payment:

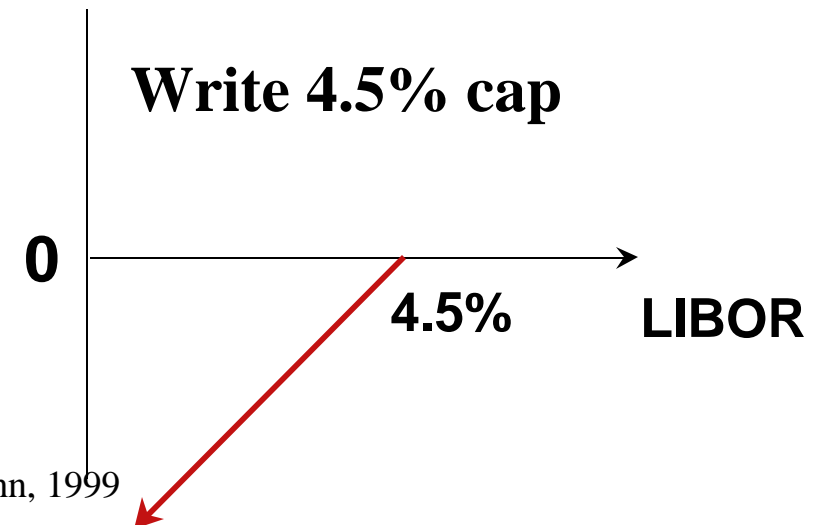
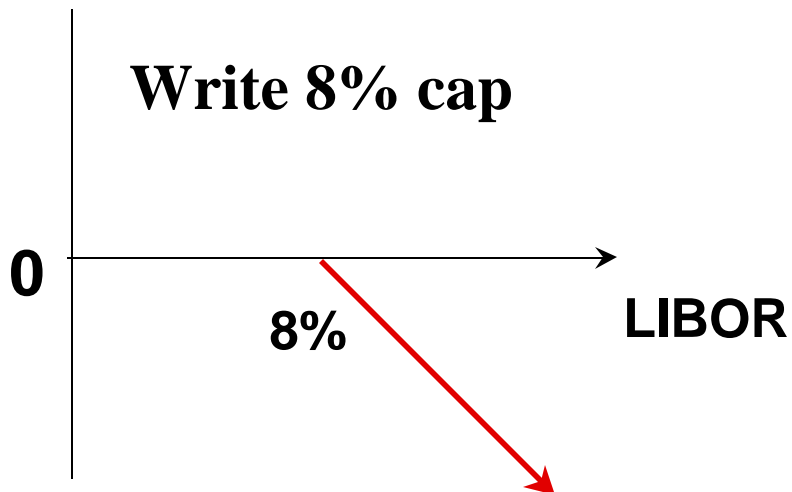
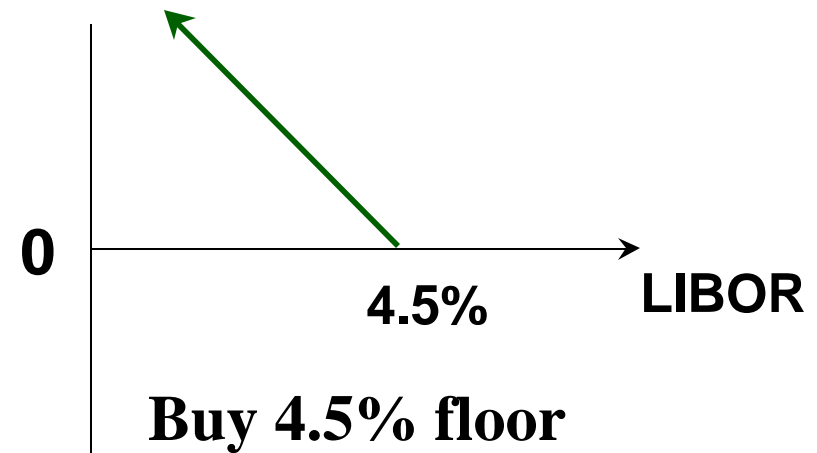
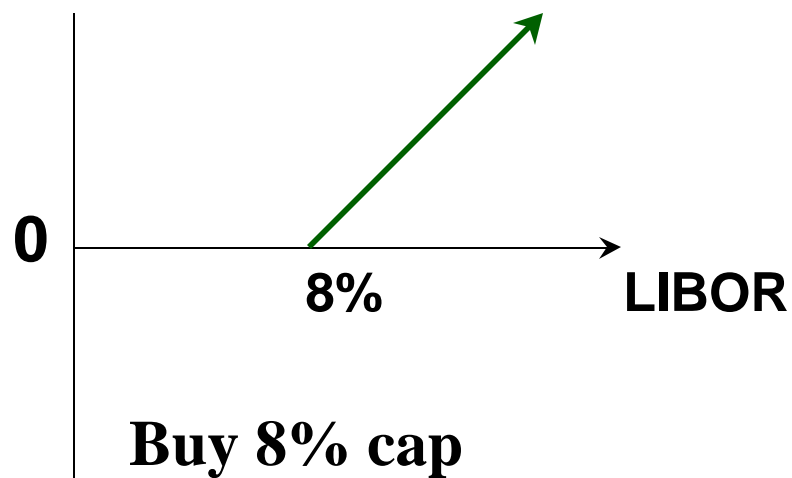
<u>if LIBOR < 4.50%</u>	<u>if LIBOR > = 4.50%</u>
$(.045 - \text{LIBOR}) \times (180/360)$	0
$\times (\text{Notional Principal})$	

example :

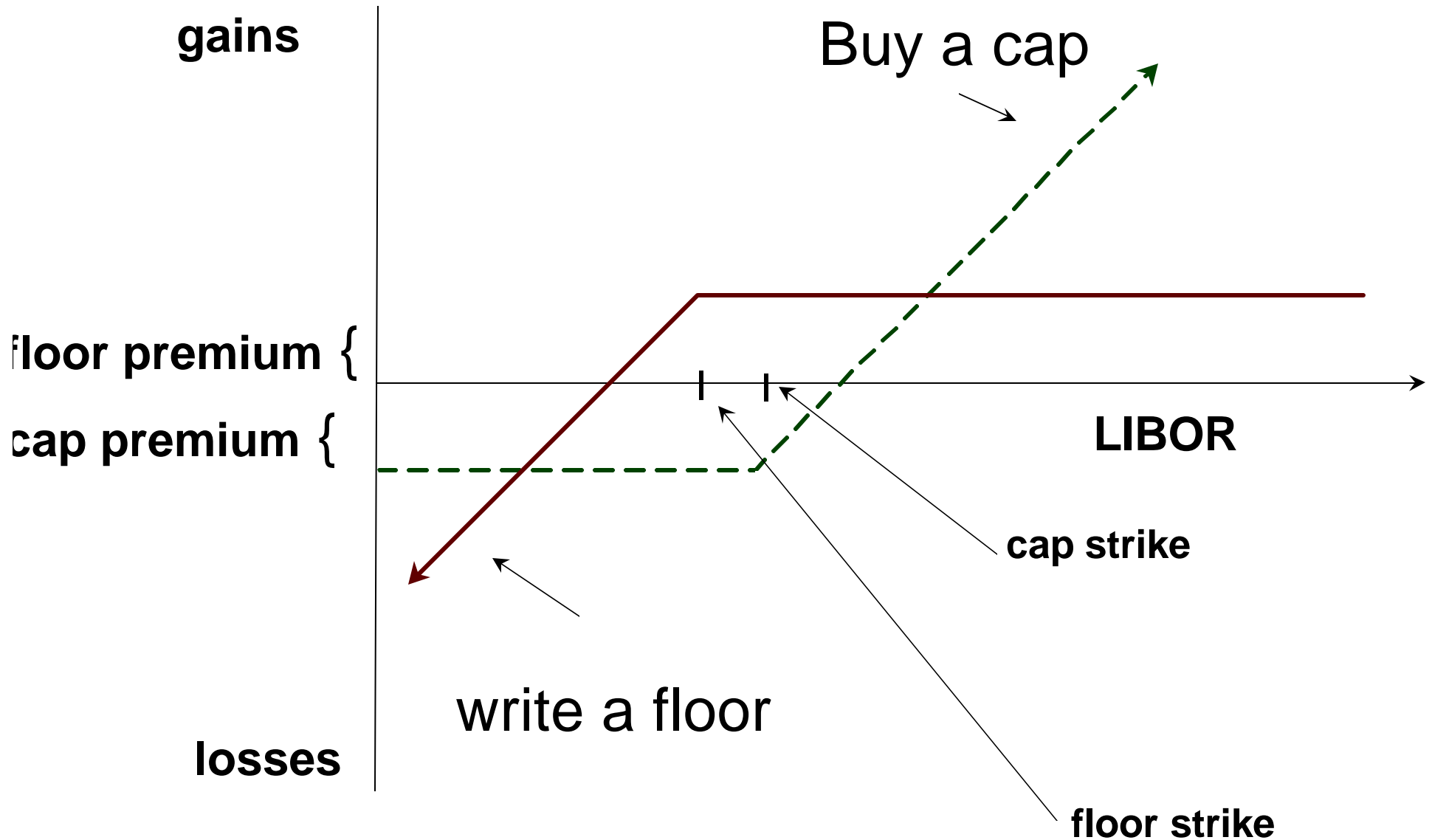
LIBOR is 4.25% at settlement, payment received
is $(.0025)(1/2)(\text{NP}) = \1250 per \$1 million
floor owner receives \$50 per million
for each basis point below the strike

Caplet and Floorlet payoffs

For any particular settlement period, the payoff for individual caplets or floorlets:

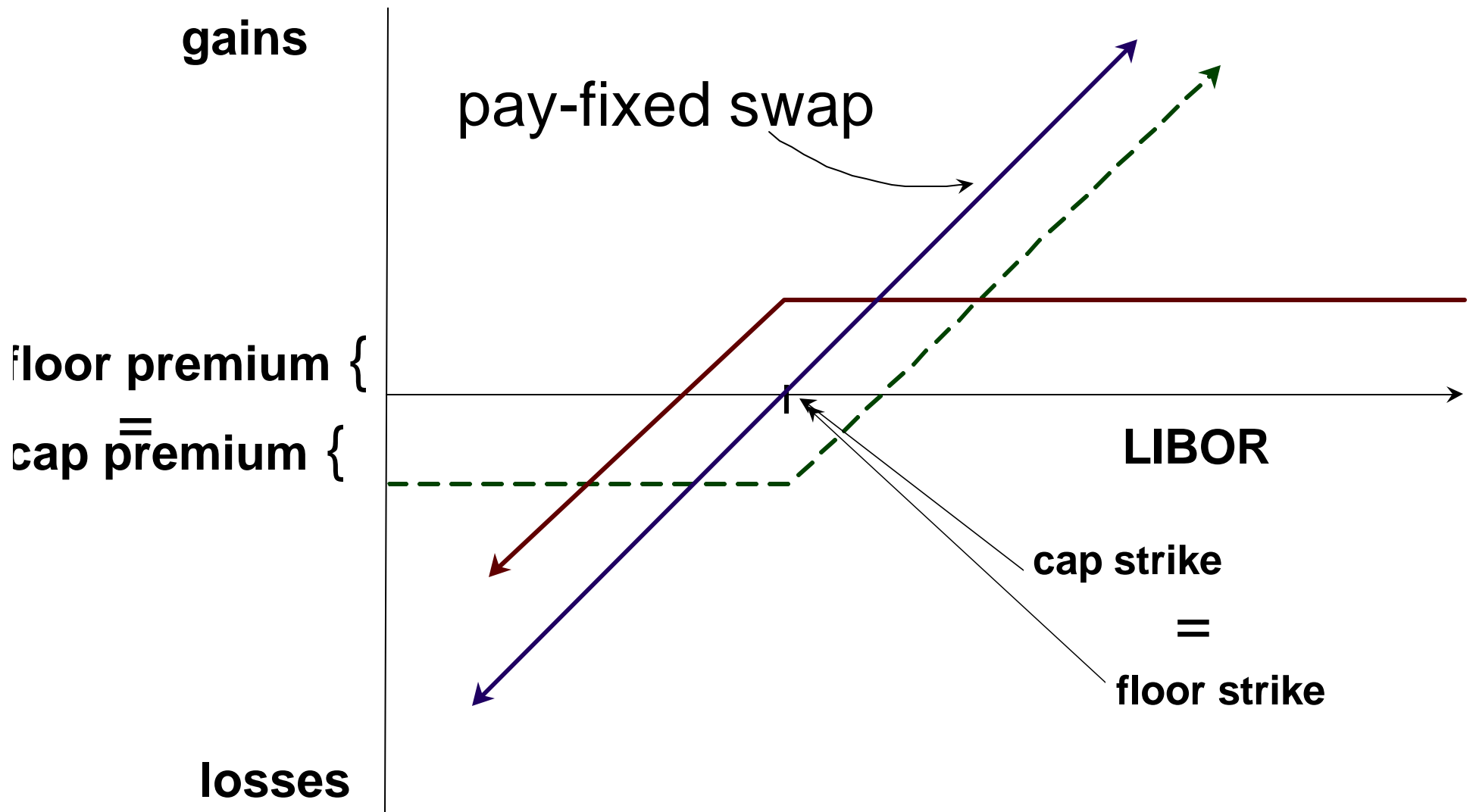


Interest rate collar



If cap premium = floor premium, it is zero-cost collar

Swap is Zero cost collar with same strike



Pay fixed swap is buying cap, writing floor to create zero-cost collar with same strikes

Swaps as options on the balance sheet

assume swap fixed rate (SFR) is 7.00 %

Swap fixed payer

Assets

Liabilities

cap
with
7%
strike

floor
with
7%
strike

swap fixed payer
buys cap and writes floor

Swap fixed receiver

Assets

Liabilities

floor
with
7%
strike

cap
with
7%
strike

swap fixed receiver
writes cap and buys floor