Hedging Strategies with Treasury Bond Futures

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Mann web page

The Chicago Board of Trade

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T-Bond Futures

- written on $100,000 face value U.S. Treasury bonds
- contract allows delivery of any bonds that are meet delivery criteria (maturity > 15 years)
- futures prices quoted in points, as a percentage of par ($100,000)
- minimum price increments of 1/32 point
- e.g. 1 point = $1000, 1/32 = $31.25
- minimum movement = $31.25
T-bond futures price quotes

- quote of 92-00 = $92,000 futures price (92% of $100k)
- if futures declines by 22/32, (contract is down 22 ticks), price declines by ($31.25) x (-22) = -$687.50, and futures price is 91-10 ($91,312.50)


- range of bonds eligible for delivery
- bonds with at least 15 year maturity
- if callable - call after 15 years

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Delivery for T-bond futures

- Short (seller) can deliver any of eligible bonds
- Eligible bonds have wide variation in coupon and maturity, thus wide variation in current price
- Short can be expected to deliver "junk" - the "cheapest -to-deliver" bonds
- Contract reflects cheapest to deliver bond
- CBOT conversion factor system used to compare bond values.

Conversion factors: [www.cbot.com](http://www.cbot.com)
Conversion factors and Futures Prices

- Conversion factors represent the price at which a given bond will yield 8%.
- Futures contract priced based on value of the "cheapest-to-deliver" bond.
- Futures price times conversion factor gives "cash equivalent price".
  - For 8% bond, conv. factor about 1.
  - Coupon < 8%, factor is less than 1.
  - Coupon > 8%, factor is more than 1.
Maturity rounded down to nearest 3 months for calculation

Consider 8.5% T-bond with maturity 22 years, 2 months. Conversion factor prices at 8%:

Bond value defined to be:

\[ \sum_{t=1}^{44} \frac{4.25}{(1 + .04)^t} + \frac{100}{(1 + .04)^{44}} = 105.1372 \]

So conversion factor is 1.0514 for the bond.
Cheapest to deliver

Short invoice is: (short receives):

\((\text{Futures price } \times \text{ bond conversion factor}) + \text{ bond accrued interest}\)

cost of purchasing bond to deliver is:

bond price + accrued interest

so, net gain to short from delivering is:

\(\text{Futures price } \times \text{ conversion factor} - \text{ bond price}\)
Example: Cheapest to deliver

<table>
<thead>
<tr>
<th>Bond</th>
<th>Price</th>
<th>Conversion factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.25</td>
<td>1.0820</td>
</tr>
<tr>
<td>2</td>
<td>126.00</td>
<td>1.4245</td>
</tr>
<tr>
<td>3</td>
<td>142.125</td>
<td>1.5938</td>
</tr>
</tbody>
</table>

Current futures price is: 94 - 2 (94.0625)

Gain to short from delivering is:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Gain from delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.0625 x 1.0820 - 94.25 = 7.53</td>
</tr>
<tr>
<td>2</td>
<td>94.0625 x 1.4245 - 126.00 = 7.99</td>
</tr>
<tr>
<td>3</td>
<td>94.0625 x 1.5938 - 142.125 = 7.79</td>
</tr>
</tbody>
</table>

Bond 2 is the cheapest to deliver.
Scenario: Pension Fund Manager will need to liquidate bonds in 40 days in order to make payment of $5 million to beneficiaries.

Risk Profile:

Value of Bonds to be sold

rates

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Duration

Define $D_M = \frac{\text{duration}}{1+y}$ (annual coupon)

= $\frac{\text{duration}}{1+y/2}$ (semi-annual coupon)

(modified duration)

approximate % change in Price:

$$\Delta P/P = - D_M \times \Delta y$$

example:

$D_M = 4.5$

$\Delta y = + 30 \text{ bp}$

expected % price change = $-4.5 \times (0.0030) = -1.35\%$

linear approximation. Convexity matters.

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Price Value of Basis Point (PVBP)

PVBP = \( D_M \times \text{Portfolio Value} \times 0.0001 \)

Example: price = $100,000; \( D_M = 4.62 \)

PVBP = \((4.62) \times 100,000 \times 0.0001\) = $46.20

PVBP for T-Bond futures:

PVBP(Bond futures) = \( \frac{\text{PVBP(Cheapest to deliver)}}{\text{Conversion factor}} \)

Example:

cheapest to deliver price = $126.00
cheapest to deliver \( D_M \) = 10.00
conversion factor = 1.4245

PVBP(futures) = \((10.0 \times 126,000 \times 0.0001) / 1.4245\)

= $88.54

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# T-Bond futures PVBP example

Assume Cheapest to deliver is 8.75% of May 15/2020

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>par value</td>
<td>100,000</td>
<td>Macauley duration</td>
<td>11.63</td>
</tr>
<tr>
<td>settlement date</td>
<td>11/23/97</td>
<td>$D_m$</td>
<td>11.28</td>
</tr>
<tr>
<td>bond maturity</td>
<td>5/15/20</td>
<td>PVBP (spot)</td>
<td>148.47</td>
</tr>
<tr>
<td>bond coupon</td>
<td>8.75%</td>
<td>maturity (years)</td>
<td>22.48</td>
</tr>
<tr>
<td>bond yield</td>
<td>6.14%</td>
<td>conversion factor</td>
<td>1.0777</td>
</tr>
<tr>
<td>coupon frequency</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>bond fair value</td>
<td>131,588</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PVBP (futures)</td>
<td>137.76</td>
</tr>
</tbody>
</table>

PVBP(futures) = \( \frac{D_m \times \text{spot price} \times 0.0001}{\text{Conversion factor}} \)

Conversion factor from CBOT

**CBOT conversion factors**

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Example: Change portfolio duration

Manager expects decline in bond yields: ⇒ increase duration.
$100 million indexed bond portfolio
Current portfolio $D_m = 4.39$ ; target $= 9.6$

Steps: 1. Find PVBP for portfolio:
$$4.39 \times \$100,000,000 \times .0001 = \$43,900.00$$
2. Find PVBP for target portfolio:
$$9.60 \times \$100,000,000 \times .0001 = \$96,000.00$$
3. Use futures PVBP (prior slide) to find futures position needed to extend duration to target
$$(\$96,000 - \$43,900)/\$137.76 = 378.2 \text{ contracts}$$

Margin requirement: CBOT margins
### Synthetic Duration extension

<table>
<thead>
<tr>
<th>Portfolio D_M</th>
<th>4.39</th>
<th>PVBP (futures)</th>
<th>137.76</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio Value</td>
<td>100,000,000</td>
<td>Portfolio PVBP</td>
<td>43,900</td>
</tr>
<tr>
<td></td>
<td></td>
<td>futures position</td>
<td>378</td>
</tr>
<tr>
<td>Target D_M</td>
<td>9.60</td>
<td>Target PVBP</td>
<td>96,000</td>
</tr>
</tbody>
</table>

#### Predicted values if yield:

<table>
<thead>
<tr>
<th></th>
<th>down 10 bp</th>
<th>up 10 bp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot portfolio</td>
<td>439,000</td>
<td>(439,000)</td>
</tr>
<tr>
<td>Futures</td>
<td>520,733</td>
<td>(520,733)</td>
</tr>
<tr>
<td>combined</td>
<td>959,733</td>
<td>(959,733)</td>
</tr>
<tr>
<td>target (D_M = 9.6)</td>
<td>960,000</td>
<td>(960,000)</td>
</tr>
</tbody>
</table>

This ignores convexity

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Example: Asset Allocation

$500 million portfolio: target allocation is 60% stock/40% bonds
Rise in equity leaves portfolio 80% stock ($400 m); 20% bonds.
Futures to synthetically allocate assets, trade actual assets later.
Data:
Stock beta = 1.0
S&P 500 Futures contract = $900,000.
Bond portfolio $D_M = 4.39 (target)
Bond futures PVBP = $137.76

Steps: 1. Find PVBP of additional bond exposure desired:
   PVBP = $100m x 4.39 x .0001 = $43,900
2. Determine number of bond futures to buy:
   $43,900/137.76 = 319 contracts.
3. Determine stock index futures to sell:
   $100,000,000/$900,000 = 111 contracts.

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