Hedging and speculative strategies using index futures

Finance 7523
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The Neeley School of Business

Short hedge: Sell Index futures - offset market losses on portfolio by generating gains on futures
Hedging with Stock Index futures

Example: Portfolio manager has well-diversified $40 million stock portfolio, with a beta of 1.22. 1% movement in S&P 500 Index expected to induce 1.22% change in value of portfolio.

Scenario: Manager anticipates "bear" market (fall in value), and wishes to hedge against possibility.

One Solution: Liquidate some or all of portfolio.

(Sell securities and place in short-term debt instruments until "prospects brighten")

- Broker Fees
- Market Transaction costs (Bid/Ask Spread)
- Lose Tax Timing Options (Forced to realize gains and/or losses)
- Market Impact Costs
- If portfolio large, liquidation will impact prices
Naive Short hedge: dollar for dollar

Assume you hedge using the June 96 S&P 500 contract, currently priced at 920.

Dollar for dollar hedge number of contracts:

\[
\frac{V_P}{V_F} = \frac{$40,000,000}{(920) ($500)} = 86.9 \text{ contracts}
\]

where:

\( V_P \) = value of portfolio
\( V_F \) = value of one futures contract

S.Mann, 1999
But: portfolio has higher volatility than S&P 500 Index (portfolio beta=1.20)
hedge initiated November 15, 1997, with Index at 900. If S&P drops 5% by 2/2/98 to 855, predicted portfolio change is 1.20 (-5%) = -6.0%, a loss of $2.4 million

<table>
<thead>
<tr>
<th>Date</th>
<th>Index</th>
<th>Spot position (equity)</th>
<th>futures position</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/15/97</td>
<td>900</td>
<td>$40,000,000</td>
<td>short 87 contracts</td>
</tr>
<tr>
<td>2/2/98</td>
<td>855</td>
<td>$37,600,000</td>
<td>futures drop 45 points</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-45</td>
<td>gain = (87)(45)(500)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-$2,400,000</td>
<td>= $1,957,500</td>
</tr>
</tbody>
</table>

Net loss = $442,500

Assumes:
1) portfolio moves exactly as predicted by beta
2) futures moves exactly with spot
Define hedge ratio (HR) as

\[
\text{Futures position} \quad \frac{\text{Spot Market Position}}{}
\]

Then the variance of returns on a position of one unit of the spot asset and HR units of futures is:

\[
\sigma_p^2 = \sigma_S^2 + \text{HR}^2 \sigma_F^2 + 2 \text{HR} \rho_{SF} \sigma_S \sigma_F
\]
Minimizing Hedged portfolio variance

Take partial derivative of variance w.r.t. HR, set = 0:

\[
\frac{\delta}{\delta HR} \left[ \sigma^2_S + HR^2 \sigma^2_F + 2 HR \rho_{SF} \sigma_S \sigma_F \right] = 2 HR \sigma^2_F + 2 \rho_{SF} \sigma_S \sigma_F
\]

\[
HR = \frac{\rho_{SF} \sigma_S \sigma_F}{\sigma^2_F} = \frac{COV_{SF}}{\sigma^2_F}
\]

This is the regression coefficient \( \beta_{SF} \)

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Estimating Minimum Variance Hedge ratio

Estimate Following regression:

\[ \text{spot return}_t = \alpha + \beta_{SF} \text{Futures ‘return’}_t + \epsilon_t \]
Calculating number of contracts for minimum variance hedging

Hedge using June 98 S&P 500 contract, currently priced at 920 \[ 920 \div 900 \cdot (1 + r - d)^t \]

Minimum variance hedge number of contracts:

\[
\frac{V_P}{V_F} \cdot \beta_{PF} = \frac{\$40,000,000}{(920) \cdot ($500)} (1.20) = 104 \text{ contracts}
\]

where: $V_P$ = value of portfolio  
$V_F$ = value of one futures contract  
$\beta_{PF}$ = beta of portfolio against futures
Minimum variance hedging results

Hedge initiated November 15, 1997, with Index at 900. If S&P drops 5% by 2/2/98 to 855, predicted portfolio change is $1.20 (-5%) = -6.0%, a loss of $2.4 million.

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<td>855</td>
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<td>futures drop 45 points</td>
</tr>
<tr>
<td></td>
<td>-45</td>
<td>-$2,400,000</td>
<td>gain = (104)(45)(500) = $2,340,000</td>
</tr>
</tbody>
</table>

Net loss = $60,000

Assumes:
1) Portfolio moves exactly as predicted by beta
2) Futures moves exactly with spot
   (note: Futures actually moves more than spot: \( F = S(1+c) \))
Altering the beta of a portfolio

Define $\beta_{PF}$ as portfolio beta (against futures)

Change market exposure of portfiolio to $\beta_{new}$ by buying (selling):

$$\frac{V_P}{V_F} \ (\beta_{new} - \beta_{PF}) \text{ contracts}$$

where:

$V_F = \text{value of one futures contract}$

$V_P = \text{value of portfolio}$
Impact of synthetic beta adjustment

Return on Portfolio

Return on Market

Original Expected exposure

Expected exposure from reducing beta

Expected exposure from increasing beta

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Changing Market Exposure: Example

Begin with $40,000,000 portfolio: $\beta_{PF} = 1.20$

Prefer a portfolio with 50% of market risk ($\beta_{New} = .50$)

Hedge using June 98 S&P 500 contract, currently priced at 920.

$$\frac{V_P}{V_F} (\beta_{New} - \beta_{PF}) = \frac{\$40,000,000}{(920) (\$500)} (0.50 - 1.20)$$

$$= -61 \text{ contracts}$$

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Reducing market exposure: results

hedge initiated November 15, 1997, with Index at 900. If S&P drops 5% by 2/2/98 to 855, predicted portfolio change is 1.20 (-5%) = -6.0%, a loss of $2.4 million. Predicted loss for $\beta_p = 0.5$ is (-2.5% of $40$ m) = $1$ million

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<td>futures drop 45 points</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-45</td>
<td>gain = (61)(45)(500)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-$2,400,000</td>
<td>= $1,372,500</td>
</tr>
</tbody>
</table>

Net loss = $1,027,500

Assumes:
1) portfolio moves exactly as predicted by beta
2) futures moves exactly with spot
   (note: futures actually moves more than spot: $F = S(1+c)$)
Portfolio manager expects sector to outperform rest of market ( $\alpha > 0$)
"Alpha Capture"

\[ \text{Expected exposure with "hot" market sector portfolio} \]

Slope = $\beta_{SF}$

= beta of sector portfolio against futures

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"Alpha Capture"

Return on Asset

Expected return on short hedge

Expected exposure with "hot" market sector

} expected gain on alpha

Return on Market

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Alpha Capture example

Assume $10 m sector fund portfolio with $\beta = 1.30$
You expect Auto sector to outperform market by 2%
Eliminate market risk with minimum variance short hedge. (using the June 98 S&P 500 contract, currently priced at 920)

The desired beta is zero, and the number of contracts to buy (sell) is:

$$\frac{V_P}{V_F} (\beta_{\text{NEW}} - \beta_{\text{SF}}) = \frac{$10,000,000}{(920) ($500)} (0 - 1.30)$$

$$= -28 \text{ contracts}$$

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Alpha capture: market down 5%

hedge initiated November 15, 1997, with Index at 900. If S&P drops 5% by 2/2/98 to 855, predicted portfolio change is $1.30(-5\%) + 2\% = -4.5\%$, a loss of $450,000.

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<thead>
<tr>
<th>Date</th>
<th>Index</th>
<th>Spot position (equity)</th>
<th>futures position</th>
<th>futures drop 45 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/15/97</td>
<td>900</td>
<td>$10,000,000</td>
<td>short 28 contracts</td>
<td></td>
</tr>
<tr>
<td>2/2/98</td>
<td>855</td>
<td>$9,550,000</td>
<td>-45</td>
<td>- $450,000</td>
</tr>
</tbody>
</table>

Net GAIN = $180,000

Assumes:
1) portfolio moves exactly as predicted by beta
2) futures moves exactly with spot
   ( note: futures actually moves more than spot : $F = S (1+c)$)
hedge initiated November 15, 1997, with Index at 900. If S&P rises 5% by 2/2/98 to 945, predicted portfolio change is $1.30 \times (5\%) + 2\% = 8.5\%$, a gain of $850,000$.

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<th>futures position</th>
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<tr>
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<td>$10,000,000</td>
<td>short 28 contracts</td>
</tr>
<tr>
<td>2/2/98</td>
<td>945</td>
<td>$10,850,000</td>
<td>futures rise 45 points</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>$850,000</td>
<td>loss = (28)(45)(500)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= $630,000</td>
</tr>
</tbody>
</table>

Net GAIN = $220,000

Assumes:
1) portfolio moves exactly as predicted by beta
2) futures moves exactly with spot
   ( note: futures actually moves more than spot : F = S (1+c))
Implied Repo rates

Define: $i_I$ as implied repo rate (simple interest).

$i_I$ is defined by:

$$f(0,T) = S(0)(1 + i_I T) ; \text{ } f(0,T) \text{ and } S(0) \text{ are current market prices.}$$

Example: asset with no dividend, storage cost, or convenience yield (example: T-bill)

Given simple interest rate $i_s$, the theoretical forward price (model price) is:

$$f(0,T) = S(0)(1 + i_s T).$$

If $i_I > i_s$: market price > model price.

If $i_I < i_s$: market price < model price.

If $i_I > i_s$: arbitrage strategy:

<table>
<thead>
<tr>
<th></th>
<th>cost now</th>
<th>value at date T</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy asset at cost $S(0)$</td>
<td>$S(0)$</td>
<td>$S(T)$</td>
</tr>
<tr>
<td>borrow asset cost</td>
<td>$-S(0)$</td>
<td>$-S(0)(1 + i_s T)$</td>
</tr>
<tr>
<td>sell forward at $f(0,T)$</td>
<td>$0$</td>
<td>$-[S(T) - f(0,T)]$</td>
</tr>
</tbody>
</table>

Total

$$0 \quad f(0,T) - S(0)(1 + i_s T) = S(0)(1 + i_I T) - S(0)(1 + i_s T) = S(0)(i_I - i_s)T > 0$$

S.Mann, 1999
### Pricing S&P 500 Index Futures

Data from 11/10/97 Investor’s Business Daily

<table>
<thead>
<tr>
<th>Spot S&amp;P 500 Index:</th>
<th>927.51</th>
</tr>
</thead>
<tbody>
<tr>
<td>dividend yield</td>
<td>1.65%</td>
</tr>
<tr>
<td>valuation date</td>
<td>11/7/97</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>eurodollar futures</th>
<th>rates</th>
<th>(1+c)^t</th>
<th>S_0 (1+c)^t</th>
<th>actual</th>
<th>S_0 (1 + c - d)^t</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/19/97</td>
<td>5.50%</td>
<td>1.0063</td>
<td>933.32</td>
<td>931.30</td>
<td>931.61</td>
</tr>
<tr>
<td>3/20/98</td>
<td>5.63%</td>
<td>1.0204</td>
<td>946.45</td>
<td>941.00</td>
<td>940.96</td>
</tr>
<tr>
<td>6/19/98</td>
<td>5.69%</td>
<td>1.0347</td>
<td>959.69</td>
<td>950.70</td>
<td>950.43</td>
</tr>
<tr>
<td>9/18/98</td>
<td>5.81%</td>
<td>1.0500</td>
<td>973.90</td>
<td>960.70</td>
<td>960.77</td>
</tr>
<tr>
<td>12/18/98</td>
<td>5.94%</td>
<td>1.0664</td>
<td>989.06</td>
<td>971.70</td>
<td>971.91</td>
</tr>
</tbody>
</table>

S.Mann, 1999
Index Futures to speculate on Interest rates

Spot S&P 500 Index: 927.51
dividend yield 1.65%
valuation date 11/15/97
predicted parallel rise in rates (basis points) 25

<table>
<thead>
<tr>
<th>futures</th>
<th>previous rates</th>
<th>if rates rise ( S_0(1+c-d)^t )</th>
<th>25 basis pts</th>
<th>change</th>
<th>contract gain</th>
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<tbody>
<tr>
<td>12/19/97</td>
<td>5.50%</td>
<td>930.83</td>
<td>931.04</td>
<td>0.21</td>
<td>$105.70</td>
</tr>
<tr>
<td>3/20/98</td>
<td>5.63%</td>
<td>940.15</td>
<td>940.93</td>
<td>0.78</td>
<td>$392.14</td>
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<tr>
<td>6/19/98</td>
<td>5.69%</td>
<td>949.59</td>
<td>950.95</td>
<td>1.36</td>
<td>$677.89</td>
</tr>
<tr>
<td>9/18/98</td>
<td>5.81%</td>
<td>959.90</td>
<td>961.84</td>
<td>1.94</td>
<td>$969.35</td>
</tr>
<tr>
<td>12/18/98</td>
<td>5.94%</td>
<td>971.01</td>
<td>973.55</td>
<td>2.54</td>
<td>$1,270.68</td>
</tr>
</tbody>
</table>

Gain from long 12/98; short 12/97 = $1,164.98
Rate speculation: rates rise and market down 30

Spot S&P 500 Index: 897.51
dividend yield 1.65%
valuation date 11/15/97
parallel rise in rates (basis points) 25

<table>
<thead>
<tr>
<th>futures</th>
<th>original $S_0(1+c-d)^t$</th>
<th>new prices</th>
<th>change</th>
<th>contract gain</th>
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</thead>
<tbody>
<tr>
<td>12/19/97</td>
<td>930.83</td>
<td>900.92</td>
<td>-29.90</td>
<td>($14,951)</td>
</tr>
<tr>
<td>3/20/98</td>
<td>940.15</td>
<td>910.50</td>
<td>-29.65</td>
<td>($14,825)</td>
</tr>
<tr>
<td>6/19/98</td>
<td>949.59</td>
<td>920.19</td>
<td>-29.40</td>
<td>($14,701)</td>
</tr>
<tr>
<td>9/18/98</td>
<td>959.90</td>
<td>930.73</td>
<td>-29.17</td>
<td>($14,586)</td>
</tr>
<tr>
<td>12/18/98</td>
<td>971.01</td>
<td>942.06</td>
<td>-28.95</td>
<td>($14,474)</td>
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</tbody>
</table>

Gain from long 12/98; short 12/97 = $477.48
### Rate speculation: rates unchanged; market up 30

**Spot S&P 500 Index:** 957.51

**dividend yield:** 1.65%

**valuation date:** 11/15/97

**parallel rise rates (basis points):** 0

<table>
<thead>
<tr>
<th>Futures</th>
<th>Previous Rates</th>
<th>$S_0(1+c-d)^t$</th>
<th>New Prices</th>
<th>Change</th>
<th>Gain</th>
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</thead>
<tbody>
<tr>
<td>12/19/97</td>
<td>5.50%</td>
<td>930.83</td>
<td>960.93</td>
<td>30.11</td>
<td>$15,054</td>
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<tr>
<td>3/20/98</td>
<td>5.63%</td>
<td>940.15</td>
<td>970.56</td>
<td>30.41</td>
<td>$15,204</td>
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<td>5.69%</td>
<td>949.59</td>
<td>980.31</td>
<td>30.71</td>
<td>$15,357</td>
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<td>959.90</td>
<td>990.95</td>
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<td>$15,524</td>
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<tr>
<td>12/18/98</td>
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<td>971.01</td>
<td>1002.41</td>
<td>31.41</td>
<td>$15,703</td>
</tr>
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</table>

**Gain from long 12/98; short 12/97 = $649.83**