Sample problem 17 – binomial digital options - solution

Examine the following two-period binomial model:

\[ S_0 = 200 \]
\[ U = 1.1 \]
\[ D = 0.92 \]
\[ R = (1+r) = 1.01 \text{ per period} \]

Present value of $1 to be received at T is \((1+r)^{-2} = 0.9803\).

<table>
<thead>
<tr>
<th>Prob Payoff</th>
<th>SUU = $242.00</th>
<th>25%</th>
<th>$1,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$220</td>
<td></td>
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<tr>
<td>S0= $200</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>SUD = $202.40</td>
<td>50%</td>
<td>$1,000,000</td>
</tr>
<tr>
<td></td>
<td>$184</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>SDD = $169.28</td>
<td>25%</td>
<td>$0</td>
</tr>
</tbody>
</table>

Use risk-neutral probabilities to find the following values:

a) Digital Call with exercise price of $200 and Q (digital amount) of $1,000,000

Digital Call Value = PV[ Expected Payoff]
\[ = 0.9803 \times [0.25 \times \$1,000,000 + 0.50 \times \$1,000,000] \]
\[ = 0.9803 \times [0.750,000] \]
\[ = \$735,225 \]

b) Digital Put with exercise price of $200 and Q (digital amount) of $1,000,000.

Digital Put Value = PV[ Expected Payoff]
\[ = 0.9803 \times [0.25 \times \$1,000,000] \]
\[ = 0.9803 \times \$250,000 \]
\[ = \$245,075 \]