Sample problem 11 – One Period Binomials - Solution.

1. A stock price is currently $50. It is known that at the end of two months it will be either $53 or $48. The price per dollar of a two-month T-bill is $\text{B}(0,2 \text{ months}) = 0.99$.

   a) What is the value of a two-month European call with a strike price of $49.00$?

   Solution:
   In order to use the risk-neutral probabilities: need $U, D$ & $R$.
   $U = \frac{53}{50} = 1.06$
   $D = \frac{48}{50} = 0.96$
   $R = \frac{1}{0.99} = 1.01$ (close enough for us)
   
   $$\pi = \frac{r-d}{u-d} = \frac{1.01-0.96}{1.06-0.96} = 0.5$$
   
   so call price = $\text{B}(0,T) \left[ \pi C_u + (1-\pi) C_d \right]$
   
   $\text{B}(0,T) = 0.99 \left[ (.5) (4) + (.5) 0 \right] = (0.99) \$2 = \$1.98$

   b) What is the value of a two-month European put with a strike price of $49.00$?

   Solution: first way: use risk-neutral probabilities:
   
   Put price = $\text{B}(0,T) \left[ \pi P_u + (1-\pi) P_d \right]$
   
   $= 0.99 \left[ (.5) 0 + (.5) \$1.00 \right]$
   
   $= \$0.495$

   second way: use put-call parity

   $$S + P = C + \text{PV(K)}$$
   
   $50 + P = 1.98 + .99(49)$
   
   $P = 1.98 + .99(49) - 50$
   
   $= 1.98 + 48.51 - 50 = 0.49$

   (the half-cent error is because we used $1.01 = 1/0.99$)
2. A stock price is currently $50. It is known that at the end of six months it will be either $60 or $42. The price per dollar of a six-month T-bill is $\text{B(0, 6 months)} = 0.94$.

A) Calculate the value of a six-month European call option on the stock with an exercise price of $48.00.

Solution: find $U, D, R$ to use risk-neutral probabilities:

\begin{align*}
U &= \frac{60}{50} = 1.20 \\
D &= \frac{42}{50} = 0.84 \\
R &= \frac{1}{0.94} = 1.0638
\end{align*}

So \[ \pi = \frac{(1.0638 - 0.84)}{(1.20 - 0.84)} = \frac{0.2238}{0.36} = 0.62175 \]

\begin{align*}
\text{Call value} &= B(0,T) \left[ \pi C_u + (1-\pi) C_d \right] \\
&= 0.94 \left[ (0.62175) \times 12 + (0.37825) \times 0 \right] \\
&= 0.94 \left[ 7.46 \right] \\
&= 7.01
\end{align*}

B) Calculate the Value of a six-month European put option on the stock with an exercise price of $52.00. Please notice that the exercise price is different than in part A.

Solution:

\begin{align*}
\text{Call value} &= B(0,T) \left[ \pi P_u + (1-\pi) P_d \right] \\
&= 0.94 \left[ (0.62175) \times 0 + (0.37825) \times 10 \right] \\
&= 0.94 \left[ 3.7825 \right] \\
&= 3.56
\end{align*}