Sample problems #6: forward rates, bond pricing: solution

For all parts, use the following information:

\[ B(0,1) = 0.95 \]
\[ B(0,2) = 0.90 \]
\[ B(0,3) = 0.86 \]
\[ B(0,4) = 0.83 \]

a) what is the three-year forward, one-year rate \( f(3,4) \)?

\[
y(0,3) = \left( \frac{1}{B(0,3)} \right)^{1/3} - 1 = \left( \frac{1}{0.86} \right)^{1/3} - 1 = 0.163^{1/3} - 1 = 5.16\%
\]
\[
y(0,4) = \left( \frac{1}{B(0,4)} \right)^{1/4} - 1 = \left( \frac{1}{0.83} \right)^{1/4} - 1 = 0.205^{1/4} - 1 = 4.77\%
\]
\[
f(3,4) = \frac{(1+y(0,4))^4}{(1+y(0,3))^3} - 1 = \frac{(1.0477)^4}{(1.0516)^3} - 1 = 3.61\%
\]

b) consider the following bonds and their prices:

bond i) 4% annual coupon, 4-year bond, priced at $97.16
bond ii) 6% annual coupon, 4-year bond, priced at $103.24

You are directed to purchase one of the two bonds. Which bond is the best value? Show your work.

\[
\text{Price of bond (i)} = B(0,1)(S4) + B(0,2)(S4) + B(0,3)(S4) + B(0,4)(S104)
\]
\[
= 0.95(S4) + 0.90(S4) + 0.86(S4) + 0.83(S104)
\]
\[
= 97.16
\]

\[
\text{Price of bond (ii)} = B(0,1)(S6) + B(0,2)(S6) + B(0,3)(S6) + B(0,4)(S106)
\]
\[
= 0.95(S6) + 0.90(S6) + 0.86(S6) + 0.83(S106)
\]
\[
= 104.24
\]

Bond (ii) is under priced. You should buy it instead of bond (i), which is fairly priced.