Sample problems #5: Yields

1. **Yield Curves.**
You observe the following zero-coupon bond yields (all default-free).

a) Find in the price per $100 Face value for the zero-coupon bonds at each maturity:

<table>
<thead>
<tr>
<th>Year</th>
<th>Yield</th>
<th>Price per $100 Face value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.00%</td>
<td>( (1.03)^{-1} = 1/1.03 = 0.97087 = 0.9709 = B(0,1) )</td>
</tr>
<tr>
<td>2</td>
<td>4.00%</td>
<td>( (1.04)^{-2} = 1/(1.04)^2 = 0.92456 = 0.9246 = B(0,2) )</td>
</tr>
<tr>
<td>3</td>
<td>5.00%</td>
<td>( (1.05)^{-3} = 1/(1.05)^3 = 0.86384 = 0.8638 = B(0,3) )</td>
</tr>
</tbody>
</table>

b) Use the prices you calculated in part (a) to determine the price per $100 Face for the following annual coupon bonds:

i) 3% coupon 1 year bond.  
   \[
   \text{Price} = B(0,1)(103) = 0.9709(103) = \$100.00
   \]

ii) 3% coupon 2 year bond.  
    \[
    \text{Price} = B(0,1)(3) + B(0,2)(103) = 0.9709(3) + 0.9246(103) = \$98.14
    \]

iii) 3% coupon 3 year bond.  
     \[
     \text{Price} = B(0,1)(3) + B(0,2)(3) + B(0,3)(103) \\
     = 0.9709(3) + 0.9246(3) + 0.8638(103) = \$94.66
     \]

c) Use the information provided in part (a) to find the following forward rates:

i) one-year forward one-year rate, \( f(1,2) \):  
   \[
   f(1,2) = \frac{[1+y(0,2)]^2}{(1+y(0,1))} - 1 = \frac{(1.04)^2}{(1.03)} - 1 = 5.01\%
   \]

ii) two-year forward one-year rate, \( f(2,3) \):  
    \[
    f(2,3) = \frac{[1+y(0,3)]^3}{(1+y(0,2))^2} - 1 = \frac{(1.05)^3}{(1.04)^2} - 1 = 7.03\%
    \]
2. (50 points) Bond prices and forward rates.
For all parts, use the following information:

\[
\begin{align*}
B(0,1) &= 0.95 \\
B(0,2) &= 0.90 \\
B(0,3) &= 0.86 \\
B(0,4) &= 0.83
\end{align*}
\]

a) what is the three-year forward, one-year rate \( f(3,4) \)?

\[
\begin{align*}
y(0,3) &= \left[ \frac{1}{B(0,3)} \right]^{\frac{1}{3}} - 1 = (1/0.86)^{(1/3)} - 1 = (1.163)^{(1/3)} - 1 = 5.16\% \\
y(0,4) &= \left[ \frac{1}{B(0,4)} \right]^{\frac{1}{4}} - 1 = (1/0.83)^{(1/4)} - 1 = (1.205)^{(1/4)} - 1 = 4.77\% \\
f(3,4) &= (1+y(0,4))^4/(1+y(0,3))^3 - 1 = (1.0477)^4/(1.0516)^3 - 1 = 3.61\%
\end{align*}
\]

b) consider the following bonds and their prices:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Description</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>i)</td>
<td>4% annual coupon, 4-year bond</td>
<td>$97.16</td>
</tr>
<tr>
<td>ii)</td>
<td>6% annual coupon, 4-year bond</td>
<td>$103.24</td>
</tr>
</tbody>
</table>

You are directed to purchase one of the two bonds. Which bond is the best value? Show your work.

\[
\begin{align*}
\text{Price of bond (i)} &= B(0,1)(\$4) + B(0,2)(\$4) + B(0,3)(\$4) + B(0,4)(\$104) \\
&= 0.95(\$4) + 0.90(\$4) + 0.86(\$4) + 0.83(\$104) \\
&= \$97.16
\end{align*}
\]

\[
\begin{align*}
\text{Price of bond (ii)} &= B(0,1)(\$6) + B(0,2)(\$6) + B(0,3)(\$6) + B(0,4)(\$106) \\
&= 0.95(\$6) + 0.90(\$6) + 0.86(\$6) + 0.83(\$106) \\
&= \$104.24
\end{align*}
\]

Bond (ii) is under priced. You should buy it instead of bond (i), which is fairly priced.